Math 1, Lecture 3
Precalculus

Sample Final

Instructions: You have three hours to complete the exam. There are ten problems, worth a total of one hundred points. You may not use any books, notes, or calculators.

Write your solutions in the space below the questions. Be sure to show your work on written questions. If you need more space use the back of the page.

Name: Solutions

UID: _____

Section: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

Problem 1. *10pts.*

Circle your answer. You do not need to show your work, and no partial credit will be given. You will only receive points if it is clear to the grader which answer you have chosen.

1. In which interval is $\ln(2)$?

   (a) $(-1, 0)$

   (b) $(0, \frac{1}{2})$

   (c) $(\frac{1}{2}, 1)$

   (d) $(1, 2)$

   (e) $(2, 3)$

2. In which interval is $\tan(100°)$?

   (a) $(-\infty, -1)$

   (b) $(-1, 0)$

   (c) $(0, 1)$

   (d) $(1, \infty)$

   (e) This number is undefined.

3. If $\sin\theta = \frac{5}{13}$ and θ is in the second quadrant, which of the following is true?

   (a) $\tan\theta = \frac{5}{12}$ and $\cos\theta = \frac{12}{13}$.

   (b) $\cot\theta = -\frac{5}{12}$ and $\sec\theta = \frac{13}{5}$.

   (c) $\cos\theta = -\frac{12}{13}$ and $\cot\theta = -\frac{12}{5}$.

   (d) $\cos\theta = -\frac{12}{13}$ and $\csc\theta = -\frac{13}{5}$.

   (e) None of the above are true.

4. If $\cos\theta = \frac{2}{3}$ and θ is in the fourth quadrant, what is $\cos(2\theta)$?

(a) $-\frac{1}{9}$

(b) $\frac{1}{9}$

(c) $\frac{1}{3}$

(d) $-\frac{1}{3}$

(e) $\frac{\sqrt{5}}{3}$

5. Which expression below is not undefined?

(a) $\ln(\cos(120°))$

(b) $\tan(\frac{\pi}{2}\cos(0))$

(c) $\sin^{-1}(e^2)$

(d) $\sec(\cot^{-1}(1))$

(e) More than one of these expressions is not undefined.

Problem 2. *10pts.*

For each of the functions described or pictured below, decide whether the function described is

1. A linear function.
2. A polynomial function.
3. A rational function.
4. An exponential function.
5. A logarithmic function.
6. A sinusoidal function.

and write the appropriate number in the box next to the function.

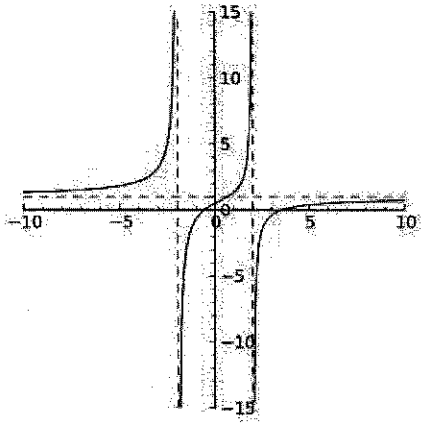
- [1] A boat is one hundred miles away from a marina, and sails directly toward it at ten miles per hour. $D(t)$ is the distance from the boat to the marina.

- [4] UCLA tuition $T(y)$ in a given year y increases by 5% each year.

- [5] Let $D(s)$ be the decibel measurement associated to a sound of intensity s. Whenever the sound becomes ten times louder, $D(s)$ increases by 20.

- [6] A water wheel completes a single revolution in ten minutes. $H(t)$ is the height of a point on the edge of the water wheel as it rotates.

- [3] The function $f(x)$ graphed here:



Problem 3.

(a) [5pts.] Simplify $\frac{1+\cot\theta}{1+\tan\theta}$ to an expression involving a single trigonometric function with no fractions.

(b) [5pts.] Prove the trigonometric identity $\sec(\beta) - \cos(\beta) = \sin(\beta)\tan(\beta)$.

(a) $\dfrac{1+\cot\theta}{1+\tan\theta} = \dfrac{1+\frac{\cos\theta}{\sin\theta}}{1+\frac{\sin\theta}{\cos\theta}}$

$= \dfrac{1}{\sin\theta}\left(\dfrac{\sin\theta+\cos\theta}{1+\frac{\sin\theta}{\cos\theta}}\right)$

$= \dfrac{1}{\sin\theta}\cdot\cos\theta\left(\dfrac{\sin\theta+\cos\theta}{\sin\theta+\cos\theta}\right)$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$

(b) $\sec B - \cos B = \dfrac{1}{\cos B} - \cos B$

$= \dfrac{1-\cos^2 B}{\cos B}$

$= \dfrac{\sin^2 B}{\cos B}$

$= \sin B \cdot \dfrac{\sin B}{\cos B}$

$= \sin B \tan B$

Problem 4.

Let $f(x) = 2\sin(2x - \pi) + 1$.

(a) [5pts.] Find the amplitude, period, and midline of $f(x)$.

(b) [5pts.] Draw a graph of $f(x)$.

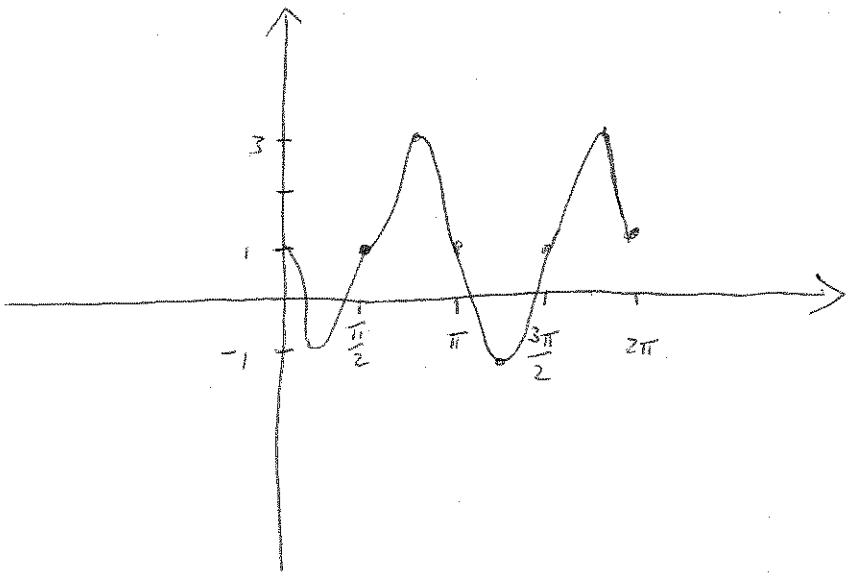
(a) Amplitude = 2

Midline is $y = 1$

Period is $\frac{2\pi}{2} = \pi$

(b) Note $f(x) = 2\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$

↑ Horizontal shift



Problem 5.

Let $f(x) = -2^{-x+1} + 4$.

(a) [5pts.] Find all asymptotes and intercepts of $f(x)$.

(b) [5pts.] Draw a graph of $f(x)$.

(a) Horizontal Asymptote $y = 4$

Vertical Intercept $y = f(0) = -2^1 + 4 = 2$

Horizontal Intercept $0 = -2^{-x+1} + 4$

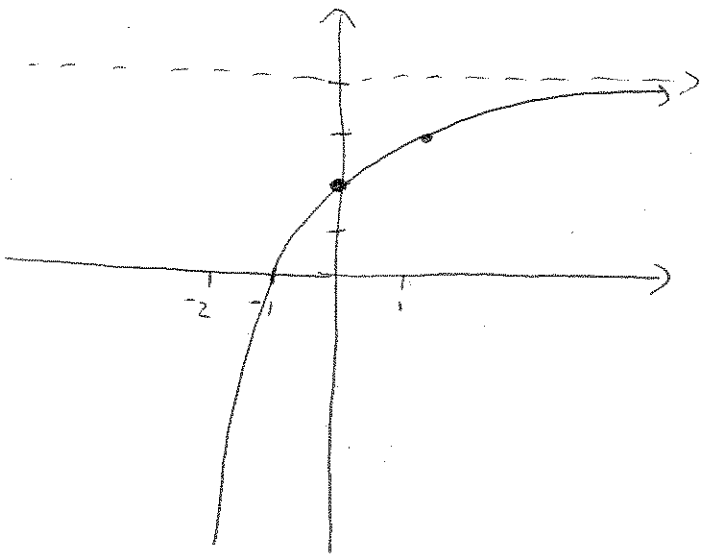
$2^{-x+1} = 4$

$-x+1 = 2$

$-x = 1$

$x = -1$

(b)



Problem 6.

Find all solutions to the following equations, or explain why there are none.

(a) [5pts.] $\sin(2\theta)\cos(3\theta) + \sin(3\theta)\cos(2\theta) = \frac{1}{2}$.

(b) [5pts.] $3^{x+2} = 4^{5x}$.

(a) $\sin(2\theta)\cos(3\theta) + \sin(3\theta)\cos(2\theta) = \sin(2\theta + 3\theta) = \sin(5\theta)$

$$\sin(5\theta) = \frac{1}{2}$$

$$5\theta = \frac{\pi}{6} + 2\pi n \quad \text{or} \quad 5\theta = \frac{5\pi}{6} + 2\pi n$$

$$\theta = \frac{\pi}{30} + \frac{2\pi n}{5} \quad \text{or} \quad \theta = \frac{\pi}{6} + \frac{2\pi n}{5}$$

(b) $3^{x+2} = 4^{5x}$

$$(x+2)\ln 3 = 5x \ln 4$$

$$x\ln 3 + 2\ln 3 = 5x \ln 4$$

$$2\ln 3 = x(5\ln 4 - \ln 3)$$

$$\frac{2\ln 3}{5\ln 4 - \ln 3} = x$$

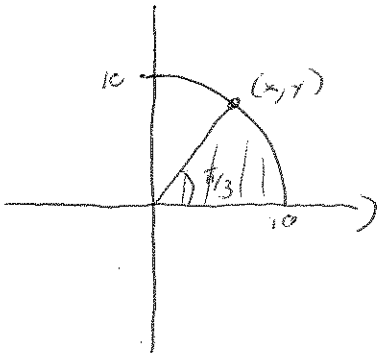
Problem 7.

Consider a circle of radius 10 centered at the origin.

(a) [5pts.] What is the area of the sector of the circle subtended by a central angle of $\frac{\pi}{3}$?

(b) [5pts.] Let the angle subtending the circle be in standard position, so that the initial side lies on the positive x-axis. What are the coordinates of the intersection of the terminal side of the angle with the circle?

(a)



$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2}(100)\left(\frac{\pi}{3}\right)$$

$$= \frac{50\pi}{3}$$

(b) $(x, y) = \left(10\cos\left(\frac{\pi}{3}\right),\ 10\sin\left(\frac{\pi}{3}\right)\right)$

$$= \left(10\left(\frac{1}{2}\right),\ 10\left(\frac{\sqrt{3}}{2}\right)\right)$$

$$= \left(5,\ 5\sqrt{3}\right)$$

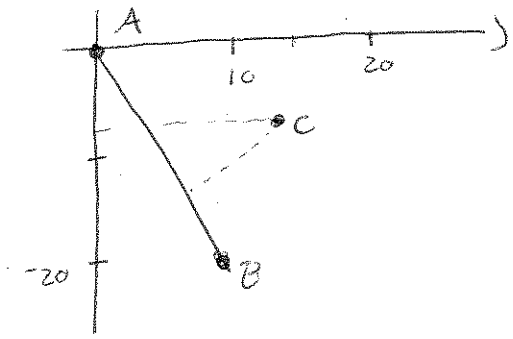
Problem 8.

You are taking a train from City A to City B, which is located 10 miles east and 20 miles south of City A. On the way you will pass near City C, which is 14 miles east and 8 miles south of City A.

(a) [5pts.] Draw a picture of this situation on a coordinate grid and find an equation for the line on which you travel to get from City A to City B.

(b) [5pts.] How close do you get to City C? [Hint: This happens when the line from you to City C is perpendicular to the line on which you are travelling.] You may leave your answer in unsimplified form.

(a)



$y = -2x$

(b) When you are closest, the line from you to City C has slope $\frac{1}{2}$ and passes through $(14, -8)$, so it is $y = \frac{1}{2}x - 15$. You pass closest to City C where these lines intersect, i.e.

$$-2x = \frac{1}{2}x - 15$$

$$-\frac{5}{2}x = -15$$

$$x = 6$$

So you are closest to City C when you are at $(6, -12)$, and the distance from you to City C is

$$\text{dist} = \sqrt{(14-6)^2 + (-8+12)^2} = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5} \text{ units}$$

Problem 9.

Decide whether the function is invertible on $\mathbb{R}$. If it is, find its inverse. If it is not, identify an interval on which it is invertible and find its inverse on that interval.

(a) [5pts.] $f(x) = 2\ln(x+3) - 5$.

(b) [5pts.] $f(x) = \frac{x-4}{2x+7}$.

(a) Invertible on its domain, $\{x : x > 3\}$ or $(3, \infty)$.

$$y = 2\ln(x+3) - 5$$

$$y + 5 = 2\ln(x+3)$$

$$\frac{y+5}{2} = \ln(x+3)$$

$$e^{\frac{y+5}{2}} = x + 3$$

$$e^{\frac{y+5}{2}} - 3 = x$$

$$f^{-1}(x) = e^{\frac{x+5}{2}} - 3$$

(b) Invertible on its domain, $\{x \neq -\frac{7}{2}\}$ or $(-\infty, -\frac{7}{2}) \cup (-\frac{7}{2}, \infty)$.

$$y = \frac{x-4}{2x+7}$$

$$2xy + 7y = x - 4$$

$$(2y-1)x = -7y - 4$$

$$x = \frac{-7y-4}{2y-1}$$

$$x = \frac{7y+4}{1-2y}$$

$$f^{-1}(x) = \frac{7x+4}{1-2x}$$

Problem 10.

Let C be a circle of radius $\sqrt{5}$ and center $(-2, 0)$

(a) [5pts.] Find an equation for C.

(b) [5pts.] At what point or points in the second quadrant does C intersect the line $y = x + 3$?

(a) $5 = (x+2)^2 + y^2$

(b) $5 = (x+2)^2 + (x+3)^2$

$5 = x^2 + 4x + 4 + x^2 + 6x + 9$

$0 = 2x^2 + 10x + 8$

$0 = 2(x^2 + 5x + 4)$

$(x+4)(x+1)$

$x = -4 \quad x = -1$

Intersection points are $(-4, 1)$ and $\boxed{(-1, 2)}$

$(-4, 1)$ ✗

↖ Lies in the second quadrant